**Exploring the Nature of Massive Star Stellar Winds with Gradual, Extended Acceleration**

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*Introduction*

Stars can reveal much about the surrounding universe. By examining the properties and characteristics associated with these stars, the mysteries of the cosmos become a little less mysterious. But not all stars are alike. Detailed studies of stellar properties help to emphasize the key differences found between stars. The main property of a star that describes how it will behave is its mass. Among the most massive of stars, which also tend to be very hot and bright, stellar winds can be quite strong. A stellar wind is the flow of particles from the surface of a star. The strength of the wind has many important implications pertaining to the evolution of the star itself, and the conditions of its local environment. Lower mass stars like the Sun have winds powered by gas pressure, while the more massive of stars have a wind dominated by radiation force.

Relative to the stars in this study, the Sun is a low mass star with a surface temperature of roughly 6000K. The surface of the Sun is surrounded by a superheated corona, reaching to temperatures above 1MK. The corona is thought to be heated by the convective processes found in the interior of the Sun. This extremely high temperature allows for gas pressure to expand and drive the outward flow of the solar wind with speeds of around

But for high-mass stars with surface temperatures ranging between 10,000K and 100,000K, the associated stellar wind is predominantly driven by radiation. This is because these stars do not have the strong convective interior needed to superheat a corona. This keeps the wind at a temperature comparable to the star’s surface temperature. With a lower temperature, the wind lacks the high gas pressure needed to drive the outward flow.

However, the radiative flux *F* that a massive star produces scales with temperature *T* as ,which comes directly from the Stefan-Boltzmann Law. This high radiative flux allows for radiation pressure to drive outflow of the stellar wind up to speeds of . This leads to an associated mass-loss-rate of up to ; or about one billion times more than the Sun’s Although a massive star only lives from ten to a few hundred million years, its mass-loss is much more significant compared to the overall mass of the star. Such large outflows of material from these massive stars contribute to the sustainability of an environment conducive for generations of stars to come.

*Electron Scattering*

Electron scattering plays an important role in the quantitative study of radiation-driven stellar winds. For a star of luminosity , the flux at a radius *r* is with the associated radiation momentum-flux simply given as:

. Scattering by electrons depend upon the opaqueness of the surrounding material. For the case of pure electron scattering, the opacity *κe* is the ratio of the Thompson cross-section to the mean atomic mass per free electron. For instance, a fully ionized gas of hydrogen with a solar mass fraction *X=0.72* has a . Considering units, the product of this opacity and radiative momentum-flux yield a force per unit mass, or an acceleration. This is the acceleration due to electron scattering, or So

Stars also produce an inward acceleration due to its gravity. This acceleration is equivalent to the simple acceleration due to gravity,

found using Newtonian mechanics.

When comparing the electron scattering acceleration to the gravitational acceleration, the ratio is spatially independent. Since both terms depend on *1/r2*, they get cancelled out. Noting this,

This is known as the Eddington Parameter; first introduced by Sir Arthur Eddington.

is known as the Eddington Limit. At this point, the star will become gravitationally unbound since the acceleration due to electron scattering is equal to that of gravity, essentially cancelling out its effect. For massive stars like the ones in this study, is on order unity with the Eddington Limit, roughly only a factor of two below. Stellar winds usually only comprise of the outermost envelope of a star, with the lower boundary representing a fixed base or arbitrary surface. This is important because if only the upper regions of the star exceed the Eddington Limit, the whole star will not become unbound. An underlying requirement made in our calculations is that the line-driving force for radiation-driven winds increases to more than that of gravity over some distance near this base.

*Recognizing a Stellar Wind*

A close up of a map

Description automatically generatedExamining a star’s light spectrum is essential when gathering information about a star. It can reveal key stellar characteristics such as temperature, luminosity, brightness, and magnitude. Looking at a star’s flux vs. wavelength profile can also uncover if there is a strong wind present. Figure 1 shows how this profile provides clear evidence for the existence of a wind. Imagine an observer on Earth examining the light produced by a distant, massive star. Photons coming from this star will appear blue-shifted or red-shifted based off of their original location relative to the stellar surface. The combined strong symmetric emission produced by the star with an associated blue-shifted absorption of photons creates a distinct profile, named the P-Cygni Profile. Blue-shifted photons indicate that they are moving towards the observer while the emission lines indicate photons coming from other parts of the star’s expanding outer shell. Recognizing a P-Cygni profile allows for the continued examination of a stellar wind.

Figure 1

<http://www.bartol.udel.edu/~owocki/RDOME/Swarthmore/sld025.htm>

*Doppler Shift Resonance for Line Scattering*

Line-scattering is the primary process that drives winds from massive stars. It is important to understand this phenomenon and how it actually causes an outflow of material, or mass-loss from a star. Line-scattering occurs when the quantum electron transitions from one discrete energy level to another. This jump causes energy shuffling or scattering to occur. The combined effect of all these energy transitions is how line-driving accelerates a wind. Noting that energy transitions only occur at a very specific wavelength, line-scattering has a highly resonant nature. Whistling is an everyday analogy used to describe this process of resonant line-scattering. Blowing air through the mouth will generate a whooshing sound that sweeps over a large number of frequencies. But when one whistles, they are generating a fine-tuned frequency that is highly amplified; very similar to what occurs when an electron changes energy state. Scattered lines have a very high energy, but only over a very small range of wavelengths.

The product of the high energies that the scattered lines create, and the number of electrons attached to an atom in a star yield a term . represents the relationship between the outward line-driving force and the inward gravitational force. For massive stars, the force due to line driving can be on order of 1000 times greater than the inward force of gravity! This says that material *could* *be* accelerated outward at fantastic speeds.

However, this is not observed in nature. This is because such a high number of atoms tuned into such a narrow frequency essentially “saturates” the line. Line saturation blocks out much of the outflowing flux, therefore reducing the associated driving force. Normally, observations will show driving forces that just barely overcome gravity; not a force on the order of 1000 times greater than gravity.

This line saturation poses a problem when attempting to study these winds. But the Doppler Effect plays a crucial role in solving this issue. Since these winds are *moving* outward, the photons are slightly redshifted which essentially de-saturates the lines. Atoms can now propagate freely at a slightly lower frequency than where they started. This now allows for a broader region of the stellar flux spectrum to be covered. With the Doppler Effect now in play, the reduction of the line force causes it to just overcome gravity and “suck” material off the star. A good analogy to describe a wind’s outward-flowing nature is to think of a straw sucking liquid out of a cup. Since the star is radiation-driven, the material on or near the surface will escape as if it is being “sucked” off. This is simply due to the fact that the line-driving force is slightly stronger than that of gravity.

*The Sobolev Length and Approximation for Line-Driving*

Stellar winds flowing from massive stars can be accelerated to speeds above 1000. Studying radiation outflow can be a challenge at such high speeds. Russian astrophysicist V. Sobolev developed an approach to better explore and understand line-transfer processes for fast winds. Recall from above that radiation has a frequency that is slightly blue-shifted as it streams out from the star. This occurs until the Doppler Effect causes the radiation frequency to be shifted to that of the line resonance. Sobolev then was able to identify the width of these resonance lines and named it a Sobolev Length:

where is the thermal speed of the outflowing material. Since the thermal speed is closely related to the sound or sonic speed, it was noted that for a supersonic wind, the Sobolev Length was on the order smaller than the variations in the flow.

Perhaps one key point to take away is that since all of the resonances are similar in nature, the conditions of the entire star can be approximated down to the local conditions at a given radius. Another important term to know is the Sobolev Optical Depth. Typically, a non-spatial integral would need to be evaluated so the optical depth calculation would cover the entirety of the star. However, with this Sobolev localization approximation, the optical depth can be reduced to terms relating to the velocity gradient and the local density of the material.

The Sobolev Optical Depth is be defined as

The Sobolev approximation allows the optical depth to be calculated only from the corresponding Sobolev Length, density, velocity gradient, and opacity located at the center of the resonance. We can further simplify this expression by introducing a *q* and *t* term:

where  
and

The *q* term helps to eliminate *vth*. The *t* term normalizes the equation to *q*=1. *q* is describing the strength of the line resonance.

Optical depth describes how much flux is limited by some amount of opacity. Similarly, the Sobolev Optical Depth aims to describe how the opacity of the flowing material reduces the intensity of the line resonances. Again, the line resonances are described by the outward flow of radiation, or by the total flux of the star.

Using this approximation is important because it allows for a simple expression for radiative acceleration to arise. In absorption processes found in massive stars, photons release momentum (*p=E/c*) to atoms, and the corresponding force is . After going through the mathematical steps, the total absorbed energy can be found as

with *dA*, *ds*, *dt,* and *dv* are changes in area, distance, time, and speed respectively.

So

where is the mass-loss-rate.

This makes

Using *grad*, the radial line acceleration is found to be

where *gthin* is the case for pure electron scattering.

In the opposite case,

where is the standard mass-loss-rate of a stellar wind. The difference between the thin and thick line acceleration is that the Sobolev Optical Depth is much less than one for the thin case and much greater than one for the thick case. The *gthick* case is particularly interesting due to the fact that it depends on the velocity gradient rather than the strength of the line like in the *gthin* case. Newton’s famous second law says that a force should *cause* an acceleration, but in the *gthick* case, the line force actually *depends* on the wind it accelerates (*vdv/dr)*.

The nature of the outflowing wind changes as the lines transition from being optically thin to optically thick. By examining (13) and (14), this becomes clear because of how different in nature the equations are.

Now is a good time to introduce a new term α which will become important in the simulations run below. α is the ratio of optically thin to optically thick lines. A higher α means there are more optically thick lines whereas a lower α suggests a higher abundance of optically thin lines. For example, if there are more optically thick lines present, the acceleration will be dominated by the velocity gradient. The nature of the acceleration is described through α. Having this term will be useful going forward when studying the cumulative line acceleration as developed and described in the CAK model below.

*CAK Line Ensemble Force Model*

Massive stars produce lines that spread over a wide range of frequencies and strengths. All such lines play a contributory role to the overall acceleration of the star’s stellar wind. Physicists Castor, Abbott, and Kline (CAK) developed a method in 1975 that describes these radial line accelerations as a cumulative whole. The overarching assumption in this model needs to be pointed out early though. This model assumes that all the lines are observed to be spectrally independent of one another, therefore keeping the spectral distribution distinct for individual lines.

Noting (12), CAK theory allows one to find the total wind acceleration

where *dN/dq* is the flux-weightednumber distribution of particles found in the wind. This can be simplified further by assuming that *dN/dq* follows a simple power law.

with representing the complete gamma function and being the total line force as mentioned above (on the order of 1000x greater than the force of gravity). Now, by applying the power law approximation for *dN/dq* and expression (15), a formal expression for the CAK line acceleration can be obtained. This expression states that

Upon closer inspection, *gCAK* represents a “geometric mean” value between the optically thin and optically thick line accelerations. Notice how (17)combines both the thin and thick cases. Knowing how to obtain *gCAK* will prove to be important in the upcoming simulation results.

*Spherically Symmetric CAK Wind Models*

For the case of a point star source producing a symmetric, radially streaming wind, utilizing Newton’s 2nd Law of Motion aids in developing a benchmark case. For a steady outflowing stellar wind, *F=ma* alters to

This simple equation of motion states that the total acceleration of the wind must equal the outward moving line acceleration minus the inward pull of gravity. In this instance, represents the reduction of gravity due to electron scattering as described above. Note that this new equation of motion does not contain a gas pressure term as it normally would for a star like the Sun. This is due to the fact that for stars like the Sun, the solar corona heats up the outflowing gas to millions of degrees. Since sound speed is associated with temperature, such a high temperature will raise the sonic speed to a value comparable to the escape speed. This therefore makes the gas pressure a necessary term in the equation of motion. However, massive stars have no such encompassing corona. So, the temperature of the outflowing wind is similar to the stellar surface temperature, which is much less than that found in the solar corona. This consequently makes the sound speed much less than the escape speed so the gas pressure term can be neglected. Gas pressure plays a negligible role in the outflow of a massive star’s wind. It now becomes clear that (18)  
has both a line-driven and a wind flow acceleration component. These components are related to one another through gravity. A new term *w’* connected to this gravitational relationship can now be introduced. This expression has a value

From here, introducing an inverse radius term

shows that . *w* represents a wind energy ratio. More specifically,

where . This *w* value is a ratio of a wind’s kinetic energy to its binding energy. Now using (17), the equation of motion can be rewritten in terms of this dimensionless quantity which yields an equation of the form

where *C* is a constant equal to

From this, it is easy to see that . This relation shows how the mass-loss is affected by the constant *C*.

It is important to now examine the critical solution to this case. Critical values *Cc*and *w’c* indicate where (21) is equal to the CAK solution. And since (21) is a nonlinear ordinary differential equation, any noncritical case will have a different number of solutions. Seeing the inverse relationship between *C* and a large mass-loss rate will have no solutions, while a small mass-loss rate will provide two solutions to (21). The critical case will only have one solution to the above ODE and is identified as the point where the mass-loss-rate is highest. Since (21) must be continuous, its derivative must also be continuous. These requirements show that

and .

Combining these critical cases together produces a result that is equivalent to the critical mass-loss-rate as defined in CAK theory where

Seeing that (21) has no spatial dependence, will hold for all radii. So, by integrating *w’* at its critical pointwith respect to the inverse radius coordinate *x*, an expression known as the beta velocity law is obtained where

and is the terminal speed defined as

with One can think of beta as a term that describes a wind’s behavior. A higher beta corresponds to a long, slow outward acceleration, while a lower beta corresponds to a shorter, quicker wind acceleration. is an interesting parameter to study since it has been claimed that it can reach extremely high values (3~4 or even higher!). Multiple observations have shown that winds can undergo slow, extended accelerations; indicating a higher .

However, in many cases, a finite disk correction factor *f* needs to be accounted for (21). *f* helps to prevent the wind from overloading with material by accounting for the star as a source with some angular dependence. With the correction factor added (21) becomes,

The first step to fully solving for *f* is to let

Note that in a pure beta-law, so, , implying that for any *1/2*, the factor *f* would become unbounded at the surface *x=0*. To correct this, the term *b* is introduced as a gravitational scaling quantity. This now sets

and by integration,

with *xc* being the critical point (equivalent to *w’c*). With this added *b* factor, the star will never become unbound unless *b=0*.

One other thing to study is that the terminal speed of the wind. When *x=1.0* in (29), the terminal speed of the wind has been reached. To find a wind’s terminal speed, the corresponding equation for *w* becomes

Notice how the terminal speed depends heavily on the location of the critical point and the value for .

Now, we are ready to move forward to our associated simulations that aim to describe the behavior of a massive star’s wind for high . We will look at characteristics such as velocity and mass-loss-rate using a hydrodynamic code called VH-1.

VH-1 Hydrodynamical Simulations

It is easy to see that there is still much that can be studied about the stellar winds from massive stars. In addition to analytic studies, numerical simulation packages provide a useful tool to help visualize and understand the processes unfolded above. In the simulations found in the following sections, a package of VH-1 hydro code developed by Stan Owocki and Steve Cranmer in the 1990s was used to produce the results.

In the simulations seen in this paper, VH-1 was updated to cancel out the correction factor f. This allows for the direct study of wind “choking” and mass overloading. Attacking this problem head-on allows us to study the wind as if it is radially streaming outward from a point source.

*VH-1 Setup*

For our simulations, we used a model star with the following input parameters.

* Wind Inflow Density:
  + Changed to

The model represents a massive spectral type OB star much larger than the Sun. We then tested our simulations over a range of values.

*The Case*

The CAK beta velocity law as described above corresponds to a . Figure 3 shows that the finite disk correction factor when *f=*1 is constantly extending outward from the stellar surface. This allows for the wind to flow away from the star following the predicted power law as shown in Figure 4. Under such conditions, the mass-loss-rate streams out at a constant rate of about 90%-95% the value of corresponding to a slowly decreasing speed roughly around the escape speed of as shown in Figure 5. Now we will examine how this ideal case changes as we increase to higher values.

Figure 3: f=1 is constant at all radii for the beta=0.5 case

Figure 4: On a logarithmic scale, this graph shows that the wind is flowing out following a simple power law when beta=0.5

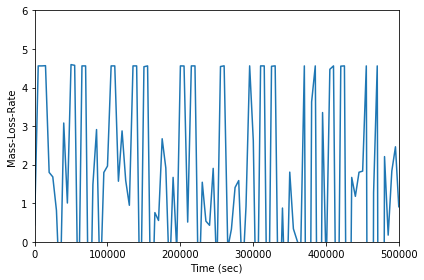
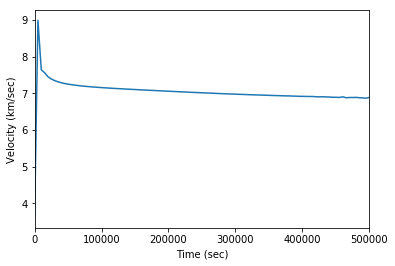
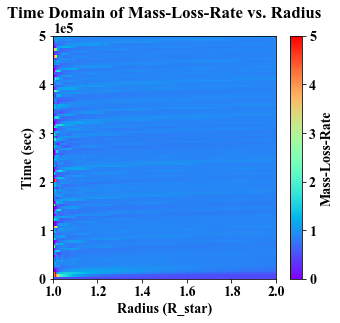
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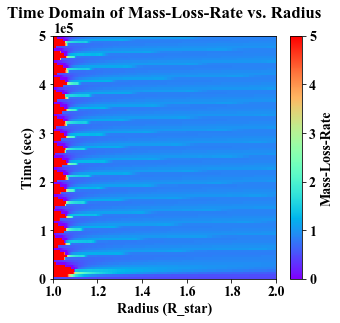
Figure 6: This figure shows the mass-loss-rate as a function of time for a beta of 0.8. Notice how there is a distinct ceiling. This ceiling implies that our wind inflow density is too small.

Figure 5: The wind speed stays fairly constant but slowly decreases over the course of the 5e5 second simulation time for beta=0.5.

With our benchmark case, we now begin to increase our value of . First, we examined the case where . This slight increase in corresponds to a wind that takes slightly longer to accelerate. From Figure 6, we can already see that the mass-loss is being “choked” or overloaded. The sharp cutoff around indicates that the associated wind inflow density (wind density at the lower boundary condition) is too small. Not all of the material can escape now. A small lower boundary wind density is one reason why it overloads so quickly.

If is too small, two things will occur in the wind. First, the wind will overload itself and cause some material to fall back down on the stellar surface. This is because the driving force can simply not suck all the material off the star at once. The other feature is that the wind will stagnate farther out from the surface. This means that the wind cannot propagate freely until it is further away. Figure 7 shows the mass-loss-rate as a function of time and radius for . One can see the how the mass-loss becomes overloaded and falls back on to the star in an oscillating pattern.

Figure 7: This figure shows the mass-loss-rate as a function of radius and time for a beta of 0.8

 This stagnation and overloading only becomes more apparent for higher . Notice in Figure 8 how, for , the mass-loss follows a clear oscillatory pattern between inflow and outflow. It also has a distinct stagnation in the wind out to a greater distance (~1.1).

To account for this enhanced stagnation and oscillatory nature, we increased by a factor of 100 to maintain this variation between outflow and inflow. Prior to increasing the wind inflow density, as we increased , the mass-loss oscillations became more saturated by taking longer to complete one oscillation. The mass would also stagnate out to further distances.

Not only does the wind density play a role in determining the behavior of the mass-loss-rate, the finite disk correction factor *f* also plays a role. Table 1 shows the nature of *f* as a function of the *x* coordinate for increasing values of . It also illustrates the associated flow speeds in terms of *w*.

Figure 8: This figure shows the mass-loss-rate as a function of radius and time for a beta of 1.0

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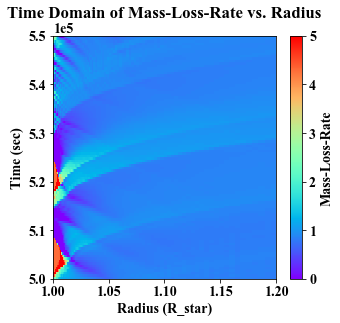
Table 1: This left figure shows the nature of f for increasing values of beta. The correction factor becomes larger and more prominent for the inner wind as beta increases. The right figure shows the associated flow speeds for differing betas. It takes longer for the wind accelerate for higher beta.

Notice how *f* gets larger in the inner wind as increases from 0.5 to 2.0. This says that there must be an initial correction factor present in order to prevent the wind from overloading in any case except for . Especially for the case where ; there is a strong *f* in the inner wind that extends outward before starting to decrease towards the critical point as shown for the other cases. Also note that *w* takes slightly longer to accelerate as increases. Not having *f* present in our simulations *and* having a slower acceleration shows how quickly the mass-loss becomes overloaded. If the associated critical point is some distance above the surface and the mass-loss becomes overloaded around this point, some material cannot escape, resulting in it falling back down to the surface. This phenomenon becomes increasingly evident as we study the mass-loss-rate for increasing values of . Table 2 shows color plots of the mass-loss-rate as a function of radius over the 500,000 second duration of the simulation for increasing values and an updated value of where .

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Table 2: A set of figures showing the oscillatory nature of the mass-loss-rate (normalized to the CAK Mass-Loss-Rate) as a function of radius for increasing beta

To ensure the validity of these results, we extended our simulation run-time by an extra 50000 seconds to ensure that these variations in flow-speed were still occurring. After running the

simulation extensions through VH-1, our results confirmed that the mass was still experiencing a change in acceleration seen in Figure 8.

But why does the mass-loss-rate oscillate between inflow and outflow at all? Recall the dimensionless ODE (21). It has a non-linear solution where the CAK critical case with one solution separates the other cases that have no and two solutions. The oscillations in mass-loss-rate show that the numerical data cannot fully relax to a constant solution as predicted by CAK theory. Rather, the solution is switching back and forth between a positive and negative acceleration.

Figure 8: Evidence for that the nature of the mass-loss-rate continues to oscillate when the simulation run-time is extended by 50000 seconds.

It is also important to look at the associated wind velocities. Table 3 shows color plots of how the wind velocities drastically increase with radius as increases. Noting from above that the associated wind terminal speed is heavily dependent on the critical point *xc* and , it can be seen that by keeping the critical point constant and increasing , the terminal speed will increase quite substantially. Table 3.a shows the associated terminal speeds for the cases studied in this paper. However, these figures in Table 3 also show “kinks” in the velocity. These “kinks” represent a velocity changing directions from outward to inward, or a switch between a positive and negative acceleration.

|  |  |
| --- | --- |
| **Terminal Speeds for** | |
| 0.8 |  |
| 1.0 |  |
| 1.2 |  |
| 1.5 |  |
| 2.0 |  |

But Tables 2 and 3 also show that the average wind mass-loss-rate and velocity for all cases relax to a nearly constant solution. By examining the outer wind (areas beyond the critical point), there seems to be an almost steady outflow of material associated with a fairly constant velocity. In these areas, we can already see that the wind has become a lot more steady-state. Looking at Table 2, the mass-loss-rate has already begun to relax to a solution around the solution in the outer wind. In Table 3, barring the changes associated with the mass-loss-rate fluctuations of the inner wind, the velocities also begin to plateau around the terminal speed in the outer wind. This eventual plateau in velocity near the terminal speed agrees with the power-law nature of the beta velocity law. Table 4 shows how each of the time averaged simulations compare to the expected results derived from the analytic beta velocity law.

Table 3.a: Beta values and their associated terminal speeds

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| /var/folders/2l/2mf9lctx0qg139xypn5bh8wc0000gn/T/com.microsoft.Word/Content.MSO/C5CECB42.tmp  Table 3: This table shows the nature of the wind velocities for increasing values of beta. | /var/folders/2l/2mf9lctx0qg139xypn5bh8wc0000gn/T/com.microsoft.Word/Content.MSO/4B194CA0.tmp |

As can be seen by the table, the time-averaged velocity values ascertained from the simulations agree quite well with the results from the analytic velocity law. Our data predicts that the velocities are slightly higher than predicted analytically. Having this slightly higher velocity corresponds to a slightly lower mass-loss-rate. This agrees nicely with a beta velocity law for the case. The nature of these winds at high values for all seem to follow a simple power law, like what is expected. This tells us that even though the wind has instabilities near the stellar surface, once it reaches a distance around critical point, it can propagate much more freely and relax to a much more steady-state solution.

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| /var/folders/2l/2mf9lctx0qg139xypn5bh8wc0000gn/T/com.microsoft.Word/Content.MSO/AD692172.tmp | /var/folders/2l/2mf9lctx0qg139xypn5bh8wc0000gn/T/com.microsoft.Word/Content.MSO/D1757250.tmp |

Table 4: Comparing the simulated time averaged velocities to the analytic beta velocity law

In general, at higher values of , stellar winds from massive stars become much more unstable in the inner regions, but, averaged out over a large radius, the wind relaxes fairly well to a standard expected value given by the beta velocity law.

*Conclusion*

There is much that we can still study regarding the winds of high-mass stars. Examining the nature of the oscillations in mass-loss-rate as done in this paper is but one of many things to explore. Future work in this area could include manipulating other key input parameters such as *xc* and/or *b*. Since the nature of the wind depends heavily on the location of the critical point, by varying its location in relation to the stellar surface, results could end up being very different in nature. Perhaps the most important parameter to manipulate would be *α*. Since *α* describes the ratio between optically thin and thick lines, the very nature of the wind will change by varying *α*. Changing *α* would subsequently change our equations for *f*, therefore altering all of the results found above when *α=0.6*.

The key findings in this paper show how the mass-loss-rate undergoes repeating times of both infalling and outflowing material. This is a result of the unstable nature of the dimensionless equation of motion in terms of *w*. The associated “kinks” in the velocity plots back up these results by periodically switching between positive and negative velocities. With a constant wind flow density *ρ* and as was increased to higher values, the wind became more overloaded since there is much more material trying to get out at once. Only so much can get out at one time. It became so diluted for and that the wind flow density needed to be raised by a factor of 10 in order to retain the oscillatory nature of the mass-loss-rate and velocity.

Studying stars and the universe around us is critical in humanity’s pursuit of knowledge. By studying how these massive stars and their winds work, scientists can gain a better understanding about how material is transported around the interstellar medium (ISM). Massive stars produce rare elements that enrich their surroundings. About 4.6 billion years ago, one such star went supernova, enriching the surrounding ISM, eventually leading to the formation of the Solar System and Earth. By studying massive stars and how they work, scientists are really trying to answer the age-old question of “where do we come from?”